Mathematical Surrealism as an Alternative to Easy-Road Fictionalism

Abstract: Easy-road mathematical fictionalists grant for the sake of argument that quantification over mathematical entities is indispensable to some of our best scientific theories and explanations. Even so they maintain we can *accept* those theories and explanations, *without believing* their mathematical components, provided we believe the concrete world is intrinsically as it needs to be for those components to be true. Those I refer to as "mathematical surrealists" by contrast appeal to facts about the intrinsic character of the concrete world, not to explain why our best mathematically imbued scientific theories and explanations are acceptable in spite of having false components, but in order to *replace* those theories and explanations with parasitic, nominalistically acceptable alternatives. I argue that easy-road fictionalism is viable only if mathematical surrealism is and that the latter constitutes a superior nominalist strategy. Two advantages of mathematical surrealism are that it neither begs the question concerning the explanatory role of mathematics in science nor requires rejecting the cogency of inference to the best explanation.

1. Introduction

Proponents of the indispensability argument for mathematical platonism contend that quantification over mathematical entities is indispensable to some of our best scientific theories and explanations. From this they infer that we ought to believe that mathematical entities exist.[1] Hard-road mathematical fictionalists such as Hartry Field resist this argument by maintaining it is possible to formulate non-parasitic, nominalistically acceptable alternatives to these theories and explanations, and thereby sensible to take a fictionalist attitude toward their mathematical

[1] For an overview of such arguments see (Colyvan 2015).

components.[2] "Hard-road" fictionalism is so named, however, because carrying out the nominalization program it requires is often thought to be difficult, if not impossible.[3]

Easy-road mathematical fictionalists, by contrast, are willing to concede (at least for the sake of argument) that it is not always possible to provide nominalistically acceptable alternatives to our best scientific theories and explanations.[4] They maintain however that it is rational to take a fictionalist attitude toward their mathematical components nonetheless. Mary Leng (2010) offers a paradigmatic version of this strategy. She argues that all it takes for a given mathematically imbued scientific theory or explanation to be a good one is for the concrete world to be intrinsically as it needs to be in order for that theory or explanation to be correct, and that mathematical entities need not exist in order for this to be the case. Accordingly she recommends *accepting* our best mathematically imbued scientific theories and explanations *without believing* all of their components (in particular without believing their mathematical components). This in turn leads her to reject the cogency of inference to the best explanation in favor of some more restricted principle (one that requires commitment only to those entities that have causal efficacy). Similar accounts have been defended by Mark Balaguer (1998; 2009), Stephen Yablo (2001; 2002; 2005; 2012), and Joseph Melia (1995; 2000; 2008).[5]

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[2] This is the program laid out in (Field 1980).

[3] See (Urquhart 1990) for reasons to doubt that Field's (1980) hard-road nominalization strategy can be successfully applied to the theory of general relativity. See (Malament 1982) for reasons to doubt that it can be successfully applied to quantum mechanics, (Balaguer 1998, ch. 6) for a response, and (Bueno 2002) for a counter response.

[4] The terminological distinction between "hard-road" and "easy-road" nominalist strategies is introduced by Colyvan (2010). While Colyvan's distinction applies to a wider class of views, this paper is concerned with the contrast insofar as it applies to versions of mathematical fictionalism.

[5] While Balaguer and Yablo both defend the feasibility of what might be classified as versions of easy-road mathematical fictionalism, they do so not in service of nominalism, but rather the view that there is no fact of the matter concerning whether mathematical entities exist.

Those I will refer to as "mathematical surrealists," by contrast, make use of a subtly different nominalist strategy.[6] Instead of appealing to facts about the intrinsic character of the concrete world in order to explain why our best mathematically imbued scientific theories and explanations are acceptable in spite of having false components, they employ those facts as a means of generating parasitic alternatives. Cian Dorr (2008, 2010) suggests, for instance, several ways that counterfactual conditionals or modal operators might be used for this purpose. One of these suggestions involves replacing a given theory, *T*, that utilizes a certain background mathematical theory, *M*, with *Necessarily, if M and the concrete realm is just as it in fact is, then T*. Another involves replacing *T* with *If M were the case and the concrete realm were just as it in fact is, then it would be the case that T*. Dorr argues that accepting such parasitic alternatives instead of their original counterparts is compatible both with embracing scientific realism and the cogency of inference to the best explanation. A similar kind of suggestion is put forward (though not advocated) by Gideon Rosen (2001). He notes that a given mathematically imbued theory, *T*, could be replaced by *T is nominalistically adequate* (where a theory is nominalistically adequate if and only if the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which that theory is true, and where the concrete core of any given world is the aggregate of all the concrete objects that exist in it).[7]

At first glance it might seem that easy-road fictionalism and mathematical surrealism are equivalent, indeed that one is a mere notational variant on the other. That there are substantive differences between them is revealed, however, by noting how each is situated in the larger

[6] The label is adapted from Leplin (1987), who coined the term 'surrealism' as a label for scientific anti-realist views that attempt to explain the success of science by positing that the non-observable portion of the world is constituted so as to make the observable portion behave as if the theories in question were true.

[7] Modal structuralist views such as those advocated by Hellman (1989) and Horgan (1984; 1987), as well as the constructibilism of Chihara (2004), might also be construed along surrealist lines. Although Hellman regards his view as a realist one, and Chihara takes a neutral stance regarding the content of mathematical theories as they are actually used.

debate surrounding the indispensability argument. As indicated above, for example, easy-road fictionalists are committed to the view that they need not believe the explanans of what they are willing to grant are examples of our best scientific explanations, which in turn commits them to denying the cogency of inference to the best explanation. Since the indispensability argument is aimed at scientific realists, however, many of whom would regard rejecting the cogency of IBE as a significant cost, this is arguably a considerable disadvantage. Mathematical surrealists, by contrast, do not share this commitment, since they are prepared to argue that IBE favors their proposed replacements.

Another dialectical cost of easy-road fictionalism pertains to the stance it must take concerning the explanatory role of mathematics. Since easy-road fictionalists argue that the mathematical explanations used in science are good even if mathematical entities do not exist, they must deny at the outset that claims about mathematical entities play an *ontologically significant* explanatory role within those explanations. But since, for advocates of explanatory versions of the indispensability argument, the question of whether mathematical entities play such a role is precisely what is at issue, this invites the charge of begging the question.[8] Mathematical surrealists, by contrast, need not take such a stance. Since they are prepared to offer alternative explanations that do not appeal to claims about mathematical entities, they might potentially grant for the sake of argument that such claims do play an ontologically significant explanatory role in the explanations they are proposing to replace.

There are also, however, at least two potential advantages that easy-road fictionalism appears to have over mathematical surrealism. First, unlike mathematical surrealism, easy-road fictionalism does not require us to *reject* some of the theories and explanations that scientists

[8] See (Baker 2009: 626-627) and (Colyvan 2010: 297).

actually use (since it allows that we may accept those theories and the relevant explanans without believing them). Thus easy-road fictionalism displays a sort of deference to working science that some might regard as more in keeping with naturalistic commitments. Second, unlike mathematical surrealism, easy-road fictionalism does not seem to require the viability of any paraphrase strategy (parasitic or otherwise).[9] If genuine, this would serve both as a dialectical and technical advantage. It would serve as a dialectical advantage because easy-road fictionalists could grant for the sake of argument a key premise of the indispensability argument (namely that quantification over mathematical entities is indispensable to some of our best scientific theories and explanations). It would serve as a technical advantage because it would avoid all the difficulties associated with mathematical surrealist paraphrase strategies. Paraphrase strategies that rely on modal operators or counterfactual conditionals like Dorr's and Rosen's have been criticized, for example (as will be discussed in more detail below), on the ground that they run afoul of the modal status of mathematical truths.

It is questionable however whether the first of these potential advantages really does provide a strong reason to prefer easy-road fictionalism to mathematical surrealism. It is open both to easy-road fictionalists and mathematical surrealists to take up Penelope Maddy's (1997: 155) observation that scientists "seem happy to use any mathematics that is convenient and effective, without concern for the mathematical existence assumptions involved," whereas scientists do not take up this same cavalier attitude when it comes to postulating new types of concrete unobservables. This feature of scientific practice affords both parties with naturalistic grounds for taking up an instrumentalist stance toward the mathematical components of the

[9] The mathematical surrealists' suggestions are "paraphrase strategies" not in the sense that they supply us with semantically or logically equivalent formulations of the original theories, but in the sense that they offer a way of taking mathematically imbued theories and systematically generating nominalistically acceptable alternatives.

scientific theories and explanations adopted by the scientific community. The two parties differ merely in the philosophical accounts they give as to why that stance is defensible. Easy-road fictionalists maintain that it is defensible on the grounds that it is reasonable to take up a fictionalist attitude toward the mathematical components of those theories and explanations. Mathematical surrealists argue that it is defensible because mathematically imbued theories and explanations can easily stand proxy for nominalistically acceptable alternatives. Mathematical surrealists, it is worth noting, might also maintain that their account accommodates the data regarding scientific practice in a way that is more in keeping with scientific realism (insofar as it fails to recommend taking up a fictionalist attitude toward what are *in fact* our best scientific theories and explanations). In either case, it is difficult to see one of these accounts as being the decisive winner on this score.

In the remainder of this paper, I argue that the second of the above potential advantages also fails to provide a good reason to prefer easy-road fictionalism to mathematical surrealism. More specifically, I argue that this alleged advantage is merely apparent, since it turns out that easy-road fictionalists are themselves committed to the viability of a mathematical surrealist paraphrase strategy. I further argue that the version of mathematical surrealism that results from adopting this strategy is preferable to easy-road fictionalism (for some of the reasons already hinted at above). If my arguments succeed, they show that easy-road fictionalism is in an interesting way self-undermining, since if it is viable, it ends up being supplanted. While this conclusion may seem hostile toward easy-road fictionalism, however, that is not entirely so. To the extent the arguments in this paper succeed, they also show that there is a nominalist strategy that is much in the same spirit as easy-road fictionalism, but which avoids some of the most significant disadvantages often thought to adhere to that view.

2. Leng's Fictionalism

While I intend my conclusions to be general, it will prove helpful to have a specific example of an easy-road fictionalist strategy in mind. Toward that end, this section summarizes both Leng's paradigmatic development of that strategy in her 2010 book (as well as in some of her previous work) and some criticisms that have been leveled against it. In subsequent sections I consider later amendments Leng makes to that account and whether they might help fortify her position.

Leng's easy-road fictionalism is aimed not merely at versions of the indispensability argument according to which quantification over mathematical entities is indispensable to the *formulation* of some of our best scientific theories, but also ones according to which it sometimes plays an *explanatory* role. One of the most discussed examples of mathematical entities allegedly playing such a role has been put forward by Alan Baker (2005; 2009; 2016) who argues that the properties of numbers figure crucially in the best explanation of why certain subspecies of North American cicadas have prime-numbered year lifecycles.[10] According to Leng's 2010 account, nominalistic scientific realists can acknowledge such examples as genuine cases of mathematical explanation within science.[11] In fact, she maintains that they can even *accept* these explanations, by way of taking a realist attitude toward what they imply about the intrinsic character of the concrete world, while taking a fictionalist attitude toward their mathematical components.

More precisely, Leng notes that the kind of mathematics employed in science can be embedded within a suitable version of set theory (Leng suggests ZFC with urelements, which

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[10] For additional cases in which mathematics is alleged to play such an explanatory role within science, see (Baker 2017a; 2017b), (Berenstain 2017), (Colyvan 1999; 2001; 2002), (Lyon and Colyvan 2008), and (Lyon 2012).

[11] Leng (2010: 10-12) characterizes her position as a version of scientific anti-realism. But she is a nominalistic scientific realist in Balaguer's (1998: 131) sense (i.e. she believes that the concrete world is intrinsically as it needs to be in order for our best scientific theories and explanations to be at least approximately correct, in spite of the fact that the abstract entities to which they are committed do not exist).

will be referred to from now on simply as "set theory"). But she does not think that we must believe that set theory is true in order to accept set-theoretically construed mathematical explanations. Rather, according to her, "the *reason* that a given mathematical explanation of a non-mathematical phenomenon is a good one is not that the mathematical utterances that make up its explanans are *true*, but rather that they are *fictional* in our make-believe of set theory with non-mathematical objects as urelements" (244).[12] That is, "the explanatory value of appeals to mathematical objects is plausibly not a result of the *existence* of such objects, but rather a result of the aptness of the *pretense* that such objects are related to non-mathematical objects in the ways our 'explanations' suppose" (244).

What makes this pretense apt? According to Leng (2010: 199-200), it is that the concrete world is intrinsically as it needs to be in order for the relevant claims about these relations between concrete and mathematical entities to be true. As she puts it in an earlier work (Leng 2005: 180 n.4), the mathematical claims appealed to in these explanations, if true, "would be true in virtue of mathematical objects being configured in a certain way and physical systems being configured in a certain way, so as to allow for the various relations posited between the mathematical and physical components to hold." "The condition imposed by the physical world" by those claims, she continues, "is, as Mark Balaguer puts it, 'that the physical world holds up its end of the "empirical-science bargain"'" (180 n.4).

From these quotes it might seem as though Leng is characterizing the aptness of the relevant pretense in terms of the truth of a certain counterfactual conditional.[13] The pretense that a certain claim, P, is true, for example, might be said to be apt just in the case that were the

[12] In the original quotation this claim is not asserted but embedded in a question. It is clear from the ensuing discussion however that Leng endorses it.

[13] This is how Baker (2009: 628) reads her 2005 account.

concrete world intrinsically just as it is and set theory true, then it would be the case that P. One commonly noted problem with this sort of characterization, however, concerns the modal status of pure mathematical theories. It is commonly held that if our standard pure mathematical theories are true at all, then they are necessarily true. If so and those theories are false (as mathematical fictionalists believe) then counterfactual conditionals that speak of what would be the case were they true have impossible antecedents. According to the standard Lewis-Stalnaker semantics for counterfactual conditionals, furthermore, all such counterfactual conditionals are only trivially true. One might attempt to circumvent this difficulty by maintaining that the existence of mathematical entities is contingent or by adopting a non-standard semantics for counterfactual conditionals. But these strategies are also not without difficulties.[14]

Leng makes it clear in her 2010 book however that she does not intend to offer such a counterfactual account (see pp. 201-207). Rather, Leng's (2010: chapter 7) notion of truth according to a pretense is taken from Kendal Walton's (1990) theory of make-believe. According to that theory, games of pretense involve the use of props together with various "rules of generation" that take facts about those props as inputs and various claims that one is licensed to pretend-true as outputs. Our use of mathematics in science, according to Leng, could be modeled as an instance of what Walton (1993) refers to as "prop oriented make-believe." When we engage in prop oriented make-believe, we do so because of our interest in the props, and not because we are interested in the content of the pretense for its own sake.[15]

The relevant prop, according to Leng's 2010 account of mathematical pretense, is nothing other than the concrete world. And the rule of generation is that we should pretend-true

[14] See (Wright and Hale 1992) for difficulties pertaining to the former strategy and (Woodward 2010) for difficulties pertaining to the latter.

[15] Yablo (2005) also appeals to Walton's account.

whatever is logically implied by the axioms of set theory when it is conjoined with true claims that are solely about the concrete world (172-181). The notion of logical implication invoked here, furthermore, is to be characterized not in terms of metaphysical possibility, but in terms of the notion of logical consistency employed in standard treatments of metalogic (the one often thought to coincide with the usual model-theoretic notion).[16] As Leng points out, a given mathematical theory might be consistent in this sense even if its truth is metaphysically impossible (205-206).

This account is consistent with Leng's earlier remarks to the effect that the explanatory value of a mathematical explanation used in science lies not in what it says about mathematical entities, but "the conditions it imposes on concrete, non-mathematical systems" (2005: 180) And since those conditions are just that the concrete world is as it intrinsically needs to be in order for the relevant mathematical claims to be true, "these conditions [could be] imposed equally well by a fictional theory as they would be by a literally true one" (180). While claims about mathematical entities may be indispensable to some our best explanations, that is merely because such appeals "provide us with an *indispensable* way of representing how things are to be taken with non-mathematical objects" (2010: 243).[17]

Baker (2009: 626-627) himself objects to this view, however, on the grounds that it "misses the point of the [Explanatory] Indispensability Argument, which is precisely to draw a sharp line between representational and explanatory uses of mathematics."[18] He also criticizes

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[16] Of course, mathematical fictionalists will not want simply to identify the relevant notion of consistency with the model-theoretic notion, since that would commit them to the existence of sets. Here Leng (2010: 56-57, 97-98) follows Field (1989: 30-38) in taking the relevant notion of consistency to be a primitive one, albeit one that extensionally coincides with the model-theoretic notion (at least for standard first-order logic) on the hypothesis that there are the sort of set-theoretic entities invoked in standard treatments of metalogic.

[17] The view that mathematics plays such a representational role is shared by Balaguer (1998; 2009), Melia (1995; 2000; 2008), and Yablo (2001; 2002; 2005; 2012).

[18] Colyvan (2010: 297) offers a similar criticism of Melia's view.

this proposal on the grounds that, while it may offer a kind of "second-order explanation" of "how mathematical explanation is possible," it "is *not* using fictionalism about mathematics to explain a physical explanandum." Baker further elaborates:

> In this situation it is crucial to distinguish between acknowledging the possible falsity of the explanans being offered and actively disbelieving the explanans while simultaneously putting it forward as an explanation. There are Moorean aspects here also. It does not seem odd for historians of science to explain why and how phlogiston could play an explanatory role in early modern theories of combustion, despite the fact that these historians do not themselves believe in the existence of phlogiston. Yet there would be something peculiar if someone were to explain combustion by appeal to phlogiston while simultaneously denying the existence of phlogiston. (627)

Another cost of Leng's 2010 proposal is that in requiring us to disbelieve components of what she is willing to grant are some of our best scientific explanations, it also requires rejecting the cogency of inference to the best explanation in favor of some more restricted principle (as Leng (2010: 218-224) acknowledges).[19] As previously indicated, this is likely by itself to make that proposal unattractive to many who are drawn to scientific realism. Mark Colyvan (2001 chapter 3: 2006: 233-234) has argued furthermore that more restricted principles than inference to the best explanation (such as inference to the most probable cause) are not sufficient to support belief in all of the various *concrete* entities to which scientific realists are committed (such as, for example, objects outside of our light cone).

3. A Surrealist Alternative

[19] Melia (2000: 472-475; 2002; 2008: 111-123) and Yablo (2012: 1022-1026) also advocate more restricted principles.

There is more that could be said about these objections. But as I will begin to argue in this section, easy-road fictionalists are committed to the viability of an alternative strategy that avoids them altogether. The strategy I have in mind is perhaps best introduced by considering yet another objection Colyvan raises against easy-road fictionalism.

Colyvan (2010) objects that easy-road fictionalism is viable only if something like the nominalization program required by hard-road fictionalism can be carried out. He does so by arguing that in order for easy-road fictionalist proposals to work, their proponents must provide an adequate sense of the nominalistic content of our mathematically imbued scientific theories and explanations (that is, of what we must believe the concrete world is like in order to accept them). And in order to do that, Colyvan argues, easy-road fictionalists must provide non-parasitic nominalistically acceptable alternatives. What is important at present is not whether Colyvan's objection succeeds, but the fact that easy-road fictionalists are committed to its failure.

Easy-road fictionalists are committed, that is, to the view that it suffices to provide an adequate sense of the nominalistic content of the mathematical components of our best scientific theories and explanations to say (as Balaguer and Leng do) that the concrete world "holds up its end of the bargain," or (less metaphorically) that the concrete world is intrinsically as it needs to be in order for the mathematical claims in question to be true according to the relevant mathematical theory. But (as I argue in the remainder) this view also commits its proponents to a systematic means of supplying parasitic alternatives to our mathematically imbued scientific theories and explanations and thereby lends itself to a mathematical surrealist strategy.

Suppose for instance that we have before us a scientific explanation of some nominalistically acceptable claim, *Q*, consisting of mathematically imbued explanans *M1*, *M2*,

… *Mn* and (perhaps) certain other nominalistically acceptable explanans *N1*, *N2*, … *Nn*. Since according to Leng's version of easy-road fictionalism, the intrinsic character of the concrete world serves to make claims that appear to be about mathematical entities correct or pretense-worthy (according to the rules of generation supplied by set theory) in the same way that, say, the actions of the cast of a play serve to make claims that appear to be about fictional people correct or pretense-worthy (according to the rules of generation supplied by the conventions of the theater), let 'CAST(…)' abbreviate 'the concrete world is intrinsically suited to be such that according to set theory …'. In light of the above we now see that easy-road fictionalists are committed to the view that the nominalistic content of each of *M1*, *M2*, … *Mn* is given by *CAST(M1)*, *CAST(M2)*, … *CAST(Mn)*.

Whereas easy-road fictionalists might appeal to this content to give a *second-order* explanation of why the mathematically imbued explanans constitute (at least part of) an acceptable explanation of *Q*, however, a mathematical surrealist might propose instead to replace *M1*, *M2*, … *Mn* of the original explanation with *CAST(M1)*, *CAST(M2)*, … *CAST(Mn)*. The resulting *alternative* explanation, they might also be prepared to argue, is a good *first-order* explanation of *Q* in its own right, one that is superior to the original on grounds of both parsimony and relevance.

This surrealist strategy will be viable provided that (i) the components of the proposed replacements are nominalistically acceptable, and (ii) the proposed replacements preserve the explanatory power of the originals. Easy-road fictionalists are not in a position to deny that the first condition is met because they themselves appeal to those components in order to state what they take to be the nominalistic content of the original explanations (although I will revisit this issue in Section 5 when I consider an objection to the effect that this content is not available to mathematical surrealists). That leaves us with the question of whether easy-road fictionalist are committed to the view that the proposed surrealist explanations meet the second condition.

One might initially think it is obvious that easy-road fictionalists are so committed. We saw, for example, that according to Leng's 2010 account, mathematics serves merely as a vehicle for representing certain facts about the concrete world, with those facts doing all the genuine explanatory work. But if that is so, one might think, it becomes hard to see how easy-road fictionalists could coherently deny that the surrealist replacements, which appeal to those facts directly, could fail to be just as explanatory as their original mathematical counterparts.

The picture is complicated, however, by the fact that in later work Leng (2012) concedes that mathematics sometimes makes explanatory contributions that go beyond how it represents the physical world. In particular, she maintains that mathematics sometimes affords what she calls "structural explanations," where these "explain a phenomenon by showing (a) that the phenomenon occurs in a physical system instantiating a general mathematical structure, and (b) the existence of that phenomenon is a consequence of the structure-characterizing axioms once suitably interpreted." (989).

There need be no concern about whether mathematical surrealists can capture the relevant content of these explanations. According to Leng (2012: 989), "We can think of a mathematical structure as characterized by axioms," and we can think of a physical system instantiating that structure as "one where those axioms are true when interpreted as about that physical system" (989). But if this is all that is involved in a physical system instantiating a mathematical structure, then either the explanans are nominalistically kosher already (because they are understood to be about concrete objects rather than abstract ones) or their nominalistic content is captured by the assertion that the concrete world is intrinsically as it needs to be in order for those explanans to be true. But capturing this nominalistic content is arguably not sufficient to do full justice to the explanatory power of these explanations. As Leng points out, part of their explanatory power lies in the fact that their explananda are shown to be *consequences* of their explanans. In other words, as Leng (2017) still later observes, part of the explanatory force of these and other sorts of mathematical explanations is *modal* in character.

If capturing the modal character of these explanations were simply a matter of stating certain logical facts about what follows from mathematical assumptions, then this would not itself pose any problem for mathematical surrealists since such purely logical facts are nominalistically acceptable already.[20] But of course, merely stating all the relevant claims about logical consequence is not enough to capture all the relevant logical relations. Even if *Q* can be shown to be logically implied by *M1*, *M2*, … *Mn*, *N1*, *N2*, … *Nn* (and we can state that it is), there is no obvious guarantee that *Q* will also be logically implied by *CAST(M1)*, *CAST(M2)*, … *CAST(Mn)*, *N1*, *N2*, … *Nn*. And insofar as preserving the explanatory power of the original mathematical explanations depends on preserving or adequately mirroring such logical relations, it is initially unclear whether the proposed surrealist replacements succeed at that task.

The above worry is an instance of a more general concern. To the extent that the proposed surrealist replacements fail to preserve or adequately mirror all of the explanatory relations that hold between the explanans and explananda of the original mathematical explanations, they might also fail to reproduce the full explanatory power the use of mathematics affords. In the next section, I begin to address this concern carrying out an investigation of the logical properties of the 'CAST(…)' operator. I do so by proposing that we understand this

[20] Or rather, they are nominalistically acceptable on the assumption that there are nominalistically kosher ways of stating metalogical claims pertaining to logical consequence. See notes 16, 29 and Section 5 for further discussion.

operator along the lines suggested by Leng's Walton-inspired account of mathematical pretense, and then use that account to formulate a truth condition for statements involving it. I then use that truth condition to argue that the proposed surrealist explanations do in fact either preserve or adequately mirror all of the relevant logical and other explanatory relations that hold between the explanans and explananda of their original mathematical counterparts. An advanced warning: In the process, I freely engage in apparent quantification over such abstract entities as theories, propositions, expression types, languages, and interpretations. But I also have more to say concerning the dialectical legitimacy of this move in Section 5.

4. Concerning the Logical and Explanatory Adequacy of Surrealist Replacements

In her own remarks about what makes a given mathematical pretense apt, Leng seems to take it for granted that set theory is not only consistent, but that it fails to imply any claims that are solely about the concrete world. This assumption may be plausible, but it is also stronger than necessary. All that is required by her account is that none of the actual facts pertaining to the intrinsic character of the concrete world are logically incompatible with set theory. That is, what is required is that there be no true claims solely about the concrete world that jointly imply the falsehood of set theory (where a given claim, *Q*, is "solely about the concrete world" if and only if necessarily, whether *Q* is true is determined solely by how the concrete world is intrinsically, independently of its relations to mathematical entities or any other abstract objects there may be).[21]

[21] Since I think we have a sufficiently intuitive grasp of what it is for a claim to be "solely about the concrete world" in this sense (and since, in any case, this is a notion that easy-road fictionalists require), I will have little by way of explicit things to say concerning which claims belong in this class. One thing that I will be assuming in the arguments that follow however is that this class is closed under possible negation. That is, I will be assuming that for any claim *P**, if *P** is solely about the concrete world, and it is possible that ~*P** is true, then ~*P** is also solely about the concrete world. I believe this assumption to be evident upon reflection. If whether *P* is true is determined solely by the intrinsic character of the concrete world, but it is possible for *P* to be false, then whether ~*P* is true must also be so determined.

Here I am speaking of Leng's proposal as though it were committed to there being relations of logical implication between *claims* (where these are perhaps most naturally thought of as propositions), whereas relations of logical implication are often thought to hold between sentences. But this need not concern us, since it is possible to define a relation of logical implication that holds between propositions (expressible in the logical language we are using) in terms of one that holds between sentences.[22] We might say, for example, that for any claims *P1**, *P2**, … *Q1**, *Q2**, … the claims *P1**, *P2**, … jointly logically imply$_P$ *Q1**, *Q2**, … if and only if there are some sentences (in the uninterpreted logical language in which we are working), S_{P1*}, S_{P2*}, … S_{Q1*}, S_{Q2*}, …, such that under some permissible interpretations of that language, S_{P1*}, S_{P2*}, … S_{Q1*}, S_{Q2*} … express *P1**, *P2**, … *Q1**, *Q2**, … (respectively), and S_{P1*}, S_{P2*}, … logically imply$_S$ S_{Q1*}, S_{Q2*} …. (from now on I will suppress subscripts indicating what type of relation of logical implication is at issue when context makes it clear).[23]

These considerations, together with Leng's proposal that we regard the rule of generation governing the relevant mathematical pretense to be that we should pretend-true whatever is logically implied by set theory conjoined with true claims solely about the concrete world, suggest the following truth condition for sentences involving the 'CAST(…)' operator:

(TC) Necessarily, for any claim *P**, *CAST(P*)* is true if and only if there are no true claims solely about the concrete world that jointly logically imply the falsity of set

[22] I will be assuming that the uninterpreted logical language with which we are working is of some standard variety (suited for mathematical applications within science) except for being supplemented by the following two additional features: First, the language somehow marks syntactically those sentences that are to be regarded as expressing propositions solely about the concrete world (with permissible interpretations constrained accordingly). Second, the 'CAST(…)' operator is to be regarded as part of the language's logical vocabulary with permissible interpretations of sentences involving it constrained by the truth condition for that operator (as stated in the next paragraph of the main text).

[23] Following Leng (2010: 48-49) (who follows Field (1989: 126)), I am thinking of the relation of logical implication that holds between sentences as corresponding to semantic consequence rather than formal deducibility.

theory, and also, there are some true claims solely about the concrete world that together with set theory jointly logically imply *P**.[24]

This truth condition allows us to see that the 'CAST(…)' operator has the following features: First, it obviously follows from *TC* that necessarily, if there are no claims solely about the concrete world that jointly logically imply the falsity of set theory, then for any claim *P**, if *P** is solely about the concrete world, then *CAST(P*)* is true. That is, we might say, the 'CAST(…)' operator *imports* any true claim that is solely about the concrete world. Second, it also follows from *TC* that the 'CAST(…)' operator is closed under conjunction. That is necessarily, for any claims *Q1** and *Q2** if *CAST(Q1*)* is true and *CAST*(*Q2**) is true, then *CAST(Q1*&Q2*)* is also true. Third, it obviously follows from *TC* that the 'CAST(…)' operator is closed under logical implication. That is, necessarily, for any claims *P** and *Q**, if *CAST(P*)* is true and *P** logically implies *Q**, then *CAST(Q*)* is also true. Finally, it can be shown that the 'CAST(…)' operator *exports* any claim to which it applies that is solely about the concrete world. That is, necessarily, for any claim *P** that is solely about the concrete world, if *CAST(P*)* is true then *P** is true.[25]

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[24] It is worth noting that this formulation does not require that we have in our grasp some *single proposition* that corresponds to set theory, which may not be the case. (There may not be such a proposition because set theory is not finitely axiomatizable, or because not all relevant instances of the axiom schemas are stateable in our vocabulary, or perhaps because the non-existence of sets renders some of the vocabulary of set theory semantically defective). Nevertheless, there will be a *class of sentences* in the uninterpreted logical language in which we are working that constitute the theorems of set theory. And we may say that some claims *P1*, *P2*, … jointly logically imply$_P$ the falsity of set theory when there are some sentences S_{P1}*, S_{P2}*, … in the language that jointly logically imply$_S$ the falsity of some of the theorems of set theory and which under some permissible interpretation express *P1**, *P2**, … (respectively). We may also say that some claims *P1**, *P2**, … *Q** together with set theory jointly logically imply$_P$ some claim *Q** when there are some sentences in the language S_{P1}*, S_{P2}*, … S_Q* which under some permissible interpretation express *P1**, *P2**, … *Q** (respectively) and S_{P1}*, S_{P2}*, … together with the theorems of set theory logically imply$_S$ S_Q*.

[25] This follows, at any rate, given the assumption I said I would be making in note 21, namely that the class of claims solely about the concrete world is closed under possible negation (i.e. for any claim *P**, if *P** is solely about the concrete world and it is possible that ~*P** is true, then ~*P** is also solely about the concrete world). *Proof:* Suppose that *P* is solely about the concrete world and that *CAST(P)* is true. Assume for reduction that ~*P* is true. Since ~*P* is true, it is possible that ~*P* is true. So since *P* is solely about the concrete world and the class of such claims is closed under possible negation, it follows that ~*P* is solely about the concrete world. Since it is true that *CAST(P)*, it follows from *TC* that there are no true claims solely about the concrete world that jointly logically imply the falsity of set theory. It also follows, however, that there are some true claims solely about the concrete world that together with set theory jointly logically imply *P*. Suppose that *Q1*, *Q2*, ... are such claims. Note that since *Q1*,

Now consider a situation in which we are replacing an explanation of a given claim, *Q*, consisting of set-theoretically imbued explanans *M1*, *M2*, … *Mn* together with nominalistically acceptable claims (solely about the concrete world) *N1*, *N2*, … *Nn* (where *Q* is a logical consequence of all these explanans) with one involving the explanans *CAST(M1)*, *CAST(M2)*, … *CAST(Mn)*, *N1*, *N2*, … *Nn*. It may be shown that this replacement explanation preserves or at least adequately mirrors the relevant logical relations as follows: Assume that each of these explanans is true. In that case, since the 'CAST(…)' operator imports any true claims that are solely about the concrete world, it follows that *CAST(N1)*, *CAST(N2)*, … *CAST(Nn)* are true. And since this operator is closed under conjunction it also follows that *CAST(M1&M2&...Mn&N1&N2&Nn)* is true. Since *M1&M2&...Mn&N1&N2&Nn* logically implies *Q* and 'CAST(…)' is closed under logical implication, furthermore, it follows that *CAST(Q)* is true. Now, if *Q* itself is not solely about the concrete world, then the fact that the explanans imply *CAST(Q)* is sufficient for nominalists, since they will deny that *Q* is a genuine fact in need of explanation (though they may accept that *CAST(Q)* is).[26] If *Q* is solely about the concrete world, however, then since the 'CAST(…)' operator exports any true claim that is solely about the concrete world, then it also follows that *Q* is true.

The above suffices to show that easy-road fictionalists like Leng should agree that surrealist explanations preserve or adequately mirror all of the logical relations that contribute to the explanatory goodness of their original mathematical counterparts. But often the explanatory

Q2, ... together with set theory jointly logically imply *P*, it follows that *Q1*, *Q2*, ... together with ~*P* jointly logically imply the falsity of set theory. But since *Q1*, *Q2*, … and ~*P* are each true claims that are solely about the concrete world, it follows from this that there are in fact some true claims solely about the concrete world that jointly logically imply the falsity of set theory, contrary to what was previously established. So by *reductio ad absurdum* we may conclude that *P* is true. By conditionalization, generalization from the arbitrary case, and necessitation, we may further conclude that necessarily, for any claim *P** that is solely about the concrete world, if *CAST(P*)* is true then *P** is true.

[26] Bangu (2008) criticizes Baker's cicada example for having an explanandum that does not pertain solely to physical(/biological) phenomena but also makes reference to mathematical entities and properties.

relations that hold among the components of a scientific explanation are not exhausted by the logical relations between them. What about probabilistic relations for example? Suppose we are considering some set-theoretically imbued explanation that employs a certain conjunction of explanans *M* to explain a certain proposition *Q*, not by virtue of implying it, but by rendering it probable.

Since easy-road fictionalists agree that it is the fact that the concrete world is intrinsically suited to stand in various relations to mathematical entities (and not the fact that it actually does so) that is relevant to whether various explanatory relations hold, they should agree in this case that *CAST(M) screens off* the probabilistic bearing of *M* on *CAST(Q)*. That is, they should agree that P(CAST(Q)|CAST(M)&M) = P(CAST(Q)|CAST(M)). But it is also a consequence of what we are taking for granted as part of our background information (namely that *M* is used in a scientific explanation and that set theory is adequate for the purposes of science) that if *M* is true then so is *CAST(M)*.[27] It follows from this last claim that P(CAST(Q)|M) = P(CAST(Q)|CAST(M)&M). And it follows from this together with the previous claim that P(CAST(Q)|M) = P(CAST(Q)|CAST(M)). If on the one hand *Q* itself is a set theoretically imbued claim, then nominalists will deny that it is a genuine truth to be explained and will seek merely to explain *CAST(Q)* instead. So in that case the fact that *CAST(M)* renders *CAST(Q)* as probable as *M* does is sufficient. On the other hand, if *Q* is a claim solely about the concrete world, then since the 'CAST(…)' operator both imports and exports all such claims, it follows from our background information that *Q* is materially equivalent to *CAST(Q)*, and so it follows

[27] It is not (non-vacuously) true in general, however, that for any given mathematical claim, *M**, if *M** is true then so is *CAST(M*)*. That will not be (non-vacuously) true, for example, for cases in which the mathematical claim in question is one concerning which set theory is silent (such as the continuum hypothesis).

that P(Q|M) = P(Q|CAST(M)).[28] Either way, easy-road fictionalists should agree that the mathematical surrealist replacement explanation preserves or mirrors the probabilistic relations that contributed to the explanatory goodness of the mathematically imbued original.

What about other explanatory relations aside from the sort of modal, logical, and probabilistic ones described above? Some scientific explanations are also causal in nature, or involve other sorts of grounding relations between the explanans and the explanandum. But since all parties are agreed that the sort of explanations at issue here are non-causal, and easy-road fictionalist are agreed that facts that pertain solely to the concrete world are not grounded in facts about actually existing mathematical entities, these sorts of explanatory relations are irrelevant. I conclude that easy-road fictionalist should agree that surrealist replacements either preserve or appropriately mirror all of the relevant modal, logical, and probabilistic relations found in their original mathematical counterparts, and that these explanatory relations exhaust all of those that are relevant.

5. Surrealism and Metalogic

One might object to the nominalistic acceptability of the arguments in the previous section on the grounds that they turn on our ability to endorse and employ *TC* which in turn involves apparent quantification over such abstract entities as claims, languages, expression types, and interpretations. But the formulation of *TC* offered above should not be regarded as occurring at the same first-order level at which the mathematical surrealist explanations are being stated. Nor should *TC* be regarded as providing semantically equivalent statements to those that employ the

[28] *Q* may not be not logically equivalent to *CAST(Q)* since there may be possible worlds in which *Q* is true but the intrinsic character of the concrete world precludes the truth of set theory. But since we are taking it for granted as part of the assumed background information that set theory is adequate for the purposes of science, we are also taking it to be part of the assumed background information that the truth of set theory is not actually so precluded.

'CAST(…)' operator at that level. Rather the above formulation should be regarded as being stated within a metalanguage that employs the apparatus of apparent quantification over abstract entities in order to point us to what the concrete world must be like intrinsically in order for the claims expressed by the relevant object-level sentences to be true. And generally speaking nominalists will need some strategy for defending the nominalistic acceptability of the kind of apparent quantification over abstract entities that occurs in metalogic.[29] So while platonists might be able to raise an objection to the mere use of such apparent quantification on the part of nominalists, this is not an objection that easy-road fictionalists are in a position to make.

Even so, these considerations do lend themselves to another sort of objection that easy-road fictionalists might raise against mathematical surrealism. They might argue that the only viable strategy available to nominalists for handling the sort of apparent quantification over abstract entities that occurs in the context of metalogic is itself an easy-road fictionalist one. That is, they might argue that in order to do metalogic, we must accept without believing certain claims pertaining to abstract objects. And if that is so (the objection goes), mathematical surrealists cannot avoid helping themselves to an easy-road fictionalist strategy.

Even if mathematical surrealists do need to rely on an easy-road strategy for purposes of doing metalogic, however, it is not clear that this consideration favors easy-road fictionalism when it comes to what nominalists should say about the use of mathematics within science. Perhaps when it comes to scientific theorizing mathematical surrealism is still superior, since it does not require its proponents to take a fictionalist attitude toward components of what they regard as our best scientific explanations, and thus accords better with scientific realism. But I

[29] Field (1989: 30-31; 1991; 1994) addresses a similar worry by employing a primitive logical implication operator in place of quantifying over model-theoretic entities and by taking a deflationary stance toward quantification over proposition-like entities. These strategies are in turn endorsed by Leng (2010: 51-57).

will not pursue this sort of response. Instead I will argue that easy-road fictionalists cannot sensibly endorse this objection in light of their own commitments.

There are two ways the present objection might be construed. The first maintains that the 'CAST(…)' operator cannot even be so much as understood along nominalistically acceptable lines without taking a fictionalist attitude toward *TC*. The second maintains that while perhaps the 'CAST(…)' operator can be understood along nominalistically acceptable lines without taking up any fictionalist attitudes, investigating its logical properties requires taking a fictionalist attitude toward *TC* and various other statements that involve quantification over abstract entities.

Let's begin our discussion of this objection by considering whether easy-road fictionalists such as Leng could endorse it on the first construal.[30] As we have already seen, according to Leng, part of what it is to adopt the attitude that a given mathematically imbued claim, *P*, is fictional according to our mathematical pretense is to believe that the concrete world is intrinsically as it needs to be for *P* to be true according set theory. That is, adopting such an attitude requires believing *CAST(P)*, where the content of this belief is to be understood in a way that renders it nominalistically acceptable. Now suppose that the only way to have such a belief is to take up a fictionalist attitude toward the sort of apparent quantification over abstract entities required to state *TC*.

What would such a fictionalist attitude involve? Presumably, this attitude rests *on the back* of yet another game of pretense, one that requires believing that the concrete world is intrinsically suited to be such that *TC* is true according to whatever theory of abstract objects we are using for the purpose of doing metalogic. So let 'CAMEL(…)' abbreviate 'the concrete

[30] It should be noted that Leng herself does not believe that an easy-road fictionalist strategy is required to handle the sort of apparent quantification over abstract entities that occurs in the context of metalogic (see note 29).

world is intrinsically suited to be such that according to the metalogical theory …'. As we have just seen, taking up the relevant fictionalist attitude toward *TC* requires believing *CAMEL(TC)*. But what does believing *CAMEL(TC)* involve? If easy-road fictionalists insist that it involves taking up yet another fictionalist attitude, they are on their way to a vicious infinite regress. If, however, they concede that 'CAMEL(TC)' can be understood along nominalistically acceptable lines without taking up such an attitude, then mathematical surrealists can take the truth conditions for sentences involving 'CAST(…)' to be supplied by *CAMEL(TC)* rather than by *TC*. And in so doing they are able to offer a surrealist construal of the truth conditions for sentences involving 'CAST(…)', contrary to what the objector maintains. Accordingly, easy-road fictionalists like Leng cannot endorse this objection given the first way of construing it.

These same considerations also show, however, why they cannot endorse this objection given the second construal. We just saw how easy-road fictionalist are committed to the view that mathematical surrealists could state the truth conditions for sentences involving 'CAST(…)' by employing *CAMEL(TC)* rather than *TC*. Likewise, any apparent quantification over abstract entities in which surrealists engage for purposes of metalogic may be regarded as in the scope of a 'CAMEL(…)' operator. Of course, at this point, easy-road fictionalists might press further by demanding that mathematical surrealists allow us to investigate the logical properties of the 'CAMEL(…)' operator itself. But the same surrealist strategy can be deployed for that purpose as well. Let 'TC'' denote the counterpart of *TC* for the 'CAMEL(…)' operator (where the former is formulated in exactly the same way as the latter *mutatis mutandis*). The surrealist can now state the truth conditions for sentences involving the 'CAMEL(…)' operator by way of employing *CAMEL*(TC') rather than TC'. If one is inclined, furthermore, to think there is something objectionably circular about this, then one should be prepared to explain why the same is not true, say, with respect to using universal quantifiers in the metalanguage in order to formulate truth conditions for sentences employing such quantifiers in the object language.

These considerations generalize to other versions of easy-road fictionalism. Melia's (2000) easy-road strategy, for instance, involves a practice that he calls "weaseling." Proponents of this strategy suggest that nominalists who wish to (quasi-)assert a given mathematically imbued claim, *P*, in the context of scientific theorizing, can "weasel" their way out of any commitment to mathematical entities they might otherwise incur, by asserting '*P* except there are no sets'. What the nominalist is doing by making such an assertion, according to Melia (2000: 467), is "taking advantage of the mathematics … to *communicate* or *express* his picture of what the world is really like." But if, by making this assertion, nominalists convey their belief about what the world is really like, then we may ask just what the content of that belief involves. What could the content of the belief *P except there are no sets* amount to except that the concrete world is intrinsically as it needs to be in order for P to be true even though there are no sets? That is, the content of the belief that the nominalist expresses in asserting '*P* except there are no sets' is just *CAST(P) and there are no sets* (or something near enough). In order to avoid inconsistency, furthermore, this content is something nominalists must be able to grasp independently of quantifying over abstract entities, unless they are able to understand such quantification by way of weaseling out of it. But on pain of a vicious infinite regress, nominalists cannot understand such quantification by way of weaseling out of quantification over abstract entities indefinitely. So they must either be able to grasp *CAST(P)* directly without weaseling or they must at least have a non-weaselly grasp of other notions in terms of which *CAST(P)* may be understood. Either way, they will have available a non-weaselly way of understanding *CAST(P)*. And for parallel reasons, they will also have a non-weaselly way of understanding the sort of apparent quantification over abstract entities they engage in for purposes of metalogic while investigating the logical properties of ‘CAST(…)’.

6. Surrealism and the Explanatory Role of Mathematics

Let’s summarize where we are. As previously noted, easy-road fictionalists grant that mathematically imbued claims occur in some of our best scientific explanations, but hold that all that is required for these explanations to be good ones is for their nominalistic content to be true. I also just argued in the previous sections that easy-road fictionalists should also grant that surrealist explanations adequately preserve or mirror all of the explanatory relations that contribute to the goodness of their original mathematical counterparts by way of invoking nothing more than what easy-road fictionalists take to be the nominalistic content of those explanations. A significant advantage of mathematical surrealism over easy-road fictionalism, furthermore, is that it does not require disbelieving components of what one takes to be examples of our best scientific explanations and thereby does not require rejecting the cogency of inference to the best explanation.

There is yet another dialectical advantage that mathematical surrealists have over easy-road fictionalists. Easy-road fictionalists and mathematical surrealists alike can accept that the mathematically imbued explanans of some scientific explanations contribute (*modulo* whether they are true) to the goodness of those explanations. But since easy-road fictionalists are committed to the acceptability of these explanations regardless of whether the mathematical entities they quantify over exist, they are committed to denying that the quantification over mathematical entities that occurs in these explanations plays any sort of *ontologically significant* explanatory role. This opens them up to the charge of begging the question against proponents of the explanatory indispensability argument, since whether quantification over mathematical entities does play such a role is precisely what is at issue. Mathematical surrealists by contrast do not share this commitment. They can concede, for the sake of argument, that these explanations are no good if their mathematical components turn out to be false. They resist the explanatory indispensability argument, not by insisting that quantification over mathematical entities does not play an ontologically significant explanatory role in some scientific explanations, but by offering explanatory alternatives.

Of course, strictly speaking, the original mathematically imbued explanations and their surrealist replacements are not incompatible with one another. In fact, necessarily, if the intrinsic character of the concrete world does not preclude the truth of set theory, and the explanans cited by the original explanations are true, then so are the explanans cited by the proposed surrealist replacements. While some might infer from this fact (together with the claim that the surrealist replacements are adequate) that the mathematical components of the originals are explanatorily idle, one might think instead that if both explanations are correct then what we have is a case of explanatory overdetermination. It might be true, that is, that the mathematical explanans cited by the former genuinely explain precisely the same facts about the intrinsic character of the concrete world as the corresponding non-mathematical explanans cited by the latter. But mathematical surrealists are also prepared to argue that since the latter require no commitment to mathematical entities and cite facts that more directly pertain to the phenomena in which we are interested, they are superior to the former, on grounds of both parsimony and relevance, and so end up being favored by inference to the best explanation.

Here one might worry that mathematical surrealism threatens to overgeneralize in a way that easy-road fictionalism does not. Insofar as nominalists are concerned with responding to the indispensability argument, it is because they want to retain a form of scientific realism (or at least a form of realism about concrete unobservables). If the mathematics employed in our best scientific explanations fails to play an ontologically significant explanatory role, as easy-road fictionalists maintain, then that is an important difference between mathematical claims and, say, claims about atoms, a difference that might explain why it is acceptable to take an anti-realist stance toward the former but not toward the latter. But if mathematics plays an ontologically significant explanatory role in science that can nevertheless be undercut by means of noting that there are surrealist alternatives, one might wonder if explanations that appeal to atoms could be undercut in a similar manner. Instead of explaining Brownian motion by way of atomic theory, for example, why not explain it instead by appealing to the fact that the observable world is as it needs to be in order for atomic theory to be true?

But here mathematical surrealists have a ready response that easy-road fictionalists are not in a position to challenge. Explanations that appeal to concrete entities such as atoms are good in part because they appeal to causal relations between observable and unobservable entities, relations that are not mirrored or reproduced by the proposed replacements. Mathematical entities by contrast do not stand in causal relations. And the logical and probabilistic relations between the explanans and explanandum that make for the goodness of these mathematical explanations are mirrored or reproduced by the mathematical surrealist replacements. So mathematical surrealists can plausibly maintain that inference to the best explanation does not favor surrealist-style explanations that dispense with quantification over concrete unobservables but does favor surrealist explanations that dispense with quantification over mathematical entities. Note, however, that while this account of why IBE does not favor mathematical explanations appeals to the acausal character of mathematical entities, mathematical surrealists are not committed to the view that IBE should be replaced with some more restricted principle that only requires us to believe our best causal explanations (or only in those entities that have causal efficacy). On the contrary, it is their view that an unrestricted version of IBE favors their proposed non-causal explanations over mathematical ones.

7. Conclusion

As foreshadowed in the introduction, the above considerations establish that easy-road fictionalism is in an interesting way self-undermining. If it is viable, it ends up supplanted by a superior strategy. But while this result undercuts easy-road fictionalism, it does not challenge the core idea around which that strategy was built. Mathematical surrealists also agree that it is sufficient to capture what is good about the mathematically imbued theories and explanations used in science to say that the concrete world is intrinsically as it needs to be for those theories or the relevant explanans to be true. The above arguments also show, furthermore, that mathematical surrealism affords a way of developing this idea that neither requires rejecting the cogency of inference to the best explanation nor begging the question concerning whether mathematics sometimes plays an ontologically significant explanatory role in science. I conclude for these reasons that mathematical surrealism is an attractive nominalist strategy and that those initially drawn to easy-road fictionalism should adopt it.[31]

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